

**MARKING GUIDE OF ORDINARY LEVEL MATHEMATICS
NATIONAL EXAMINATION 2022-2023**

SECTION A: Answer all questions

1) $7.2 \times (3.75^2 - 1.25^2) = 7.2 \times [(3.75 - 1.25)(3.75 + 1.25)] = 7.2 \times [(2.5) \times (5)]$
 $= 90$

2) $\frac{x^2-4x+4}{x^2-4} = \frac{(x-2)(x-2)}{(x-2)(x+2)} = \frac{x-2}{x+2}$

3) $\frac{5}{\sqrt{3}-\sqrt{2}} = \frac{5(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = \frac{5\sqrt{3}+5\sqrt{2}}{3-2} = 5\sqrt{3} + 5\sqrt{2}$

4) $\vec{u} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -7 \\ -13 \end{pmatrix}$

a) $\frac{1}{2}(\vec{u} + \vec{v}) = \frac{1}{2}\left[\begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} -7 \\ -13 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} -2 \\ -10 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 \\ -5 \end{pmatrix}$

b) $4\vec{u} - \vec{v} = 4\begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} -7 \\ -13 \end{pmatrix} = \begin{pmatrix} 20 \\ 12 \end{pmatrix} - \begin{pmatrix} -7 \\ -13 \end{pmatrix} = \begin{pmatrix} 20+7 \\ 12+13 \end{pmatrix} = \begin{pmatrix} 27 \\ 25 \end{pmatrix}$

5) $4x - \frac{3x+1}{2} - \frac{7x+9}{3} = 0$

$$\frac{6(4x)-3(3x+1)-2(7x+9)}{6} = 0$$

$$\frac{24x-9x-3-14x-18}{6} = 0$$

$$\frac{x-21}{6} = 0$$

$$x - 21 = 0$$

$$x = 21$$

6) $\begin{cases} 3x - 6y = 24 & (1) \\ 5x + 4y = 12 & (2) \end{cases}$

Multiply equation (1) by 2 and equation (2) by 3:

We get:

$$\begin{cases} 6x - 12y = 48 & (3) \\ 15x + 12y = 36 & (4) \end{cases}$$

Adding equation (3) and (4) we get:

$$21x = 84$$

$$\frac{21x}{21} = \frac{84}{21}$$

$$x = 4$$

Replacing the value of x in the equation (1) we get:

$$3 \times 4 - 6y = 24$$

$$12 - 6y = 24$$

$$-6y = 24 - 12$$

$$-6y = 12$$

$$6y = -12$$

$$y = \frac{-12}{6} = -2$$

$$\begin{aligned} 7) \quad \frac{5}{6}p^2q - 2p + q - 4r &= \frac{5}{6}(12)^2(7) - 2(12) + 7 - 4(9) \\ &= \frac{5}{6} \times 144 \times 7 - 24 + 7 - 36 = 840 + 7 - 60 = 787 \end{aligned}$$

$$\begin{aligned} 8) \quad a + 35^\circ &= 59^\circ \\ a &= 59^\circ - 35^\circ = 24^\circ \\ c &= a = 24^\circ \text{ Subtended by the same arc} \end{aligned}$$

$$b = c = 24^\circ \text{ (Inscribed angle subtended by the same arc)}$$

$$d + 35^\circ + a + 59^\circ = 180^\circ \text{ (Cyclic quadrilateral)}$$

$$d + 35^\circ + 24^\circ + 59^\circ = 180^\circ$$

$$d = 180^\circ - 35^\circ - 24^\circ - 59^\circ$$

$$d = 62^\circ$$

$$9) \quad \frac{AD}{AE} = \frac{CD}{BE}$$

$$\frac{12}{10} = \frac{6}{BE}$$

$$12BE = 60$$

$$BE = \frac{60}{12}$$

$$BE = 5$$

$$10) \quad 235_x = 95_{10}$$

$$2x^2 + 3x + 5 = 95$$

$$2x^2 + 3x + 5 - 95 = 0$$

$$2x^2 + 3x - 90 = 0$$

$$2x^2 + 15x - 12x - 90 = 0$$

$$x(2x + 15) - 6(2x + 15) = 0$$

$$(2x + 15)(x - 6) = 0$$

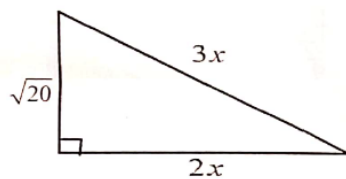
$$2x + 15 = 0 \text{ or } x - 6 = 0$$

$$x = -\frac{15}{2} \text{ (rejected) or } x = 6$$

$$11) a) I = \frac{P \times R \times T}{100 \times 12} = \frac{720,000 \times 15 \times 9}{100 \times 12} = 81,000$$

$$b) \text{ Amount of investment after 9 months} = P + I \\ = 720,000 \text{Frw} + 81,000 \text{Frw} = 801,000 \text{Frw}$$

12)



$$(3x)^2 = (2x)^2 + (\sqrt{20})^2$$

$$9x^2 = 4x^2 + 20$$

$$9x^2 - 4x^2 = 20$$

$$5x^2 = 20$$

$$x^2 = \frac{20}{5} = 4$$

$$x = \pm\sqrt{4}$$

$$x = \pm 2$$

$$x = 2 \text{ (} x = -2 \text{ rejected)}$$

$$13) a) p(A \cup B) = p(A) + p(B)$$

$$b) p(A \cap B) = p(A) \times p(B)$$

14) Let x be the smallest integer

Let $x + 1$ be the middle integer
 Let $x + 2$ be the largest integer

$$x + x + 1 + x + 2 = 21$$

$$3x + 3 = 21$$

$$3x = 21 - 3$$

$$3x = 18$$

$$x = \frac{18}{3}$$

$$x = 6$$

These integers are: The smallest integer is $x = 6$
 The middle integer is $x + 1 = 7$
 The largest integer is $x + 2 = 8$

15) $f(x) = 5x$ and $g(x) = 2x - 2$,

a) $f \circ g(x) = 5(2x - 2) = 10x - 10$

b) $g \circ f(x) = 2(5x) - 2 = 10x - 2$

SECTION B: Attempt only three questions

16) a) Frequency table

Marks (x_i)	Frequency, f_i	$x_i f_i$	Cumulative frequency
40	2	80	2
55	3	165	5
62	6	372	11
64	4	256	15
68	4	272	19
70	2	140	21
79	1	79	22
90	2	180	24
98	1	98	25
	$\sum f_i = 25$	$\sum x_i f_i = 1642$	

b) Median = $\frac{x_{n+1}}{2} = \frac{x_{25+1}}{2} = x_{13} = 64$

c) Mode = 62

$$\text{d) Mean mark} = \frac{\sum x_i f_i}{\sum f_i} = \frac{1642}{25} = 65.68$$

$$17) P(x) = 2x^3 + ax^2 + bx + 6$$

$P(x)$ is divisible by $x - 2$ if and only if $P(2) = 0$

$$x - 2 = 0 \implies x = 2$$

$$P(2) = 0; 2(2)^3 + a(2)^2 + 2b + 6 = 0$$

$$16 + 4a + 2b + 6 = 0$$

$$4a + 2b = -22$$

$$2a + b = -11$$

$$x + 1 = 0 \implies x = -1$$

$$P(-1) = 2(-1)^3 + a(-1)^2 + b(-1) + 6 = -12$$

$$P(-1) = -2 + a - b + 6 = -12$$

$$a - b = -16$$

$$\begin{cases} a - b = -16 & (1) \\ 2a + b = -11 & (2) \end{cases}$$

Adding equation (1) and (2) we get:

$$3a = -27$$

$$a = -9$$

$$a - b = -16$$

$$-9 - b = -16$$

$$-b = -7$$

$$b = 7$$

$$\text{b) } P(x) = 2x^3 - 9x^2 + 7x + 6$$

$$\text{As } x - 2 = 0 \implies x = 2$$

$$\begin{array}{c|ccc|c}
 x = 2 & 2 & -9 & 7 & 6 \\
 & \downarrow & & & \\
 & 2 & -5 & -3 & -6 \\
 \hline
 & 2 & -5 & -3 & 0
 \end{array}$$

$$P(x) = 2x^3 - 9x^2 + 7x + 6 = (x - 2)(2x^2 - 5x - 3)$$

$$h(x) = 2x^2 - 5x - 3$$

$$h(3) = 2(3)^2 - 5(3) - 3 = 18 - 15 - 3 = 0$$

$$\begin{array}{c|cc|c}
 x = 3 & 2 & -5 & -3 \\
 & \downarrow & & \\
 & 2 & 6 & 3 \\
 \hline
 & 2 & 1 & 0
 \end{array}$$

$$h(x) = 2x^2 - 5x - 3 = (x - 3)(2x + 1)$$

$$P(x) = 2x^3 - 9x^2 + 7x + 6 = (x - 2)(2x^2 - 5x - 3) = (x - 2)(x - 3)(2x + 1)$$

$$c) P(x) = 0$$

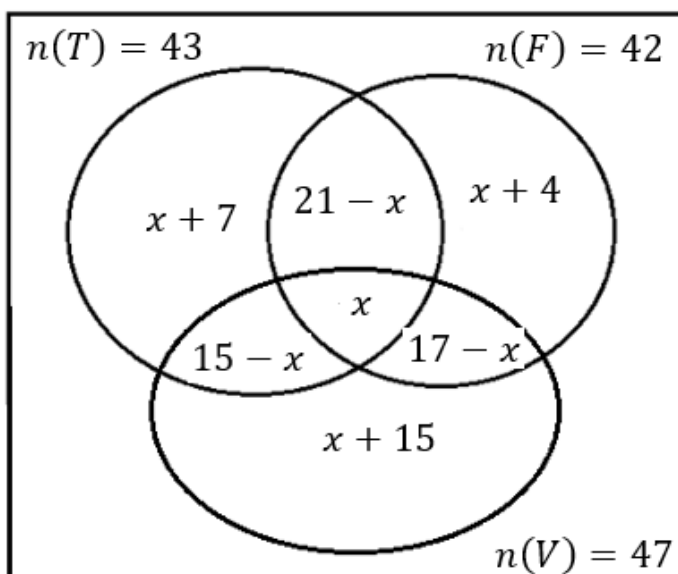
$$(x - 2)(x - 3)(2x + 1) = 0$$

$$x - 2 = 0 \text{ or } x - 3 = 0 \text{ or } 2x + 1 = 0$$

$$x = 2 \text{ or } x = 3 \text{ or } x = -\frac{1}{2}$$

$$S = \left\{-\frac{1}{2}, 2, 3\right\}$$

18) a)



$$n(\text{T}) \text{ only} = 43 - (15 - x) - (21 - x) - x = x + 7$$

$$n(\text{F}) \text{ only} = 42 - (21 - x) - (17 - x) - x = x + 4$$

$$n(\text{V}) \text{ only} = 47 - (15 - x) - (17 - x) - x = x + 15$$

$$\text{b) } x + 7 + 15 - x + x + 21 - x + x + x + x + 4 + 17 - x + x + 15 = 87$$

$$x + 79 = 87$$

$$x = 87 - 79$$

$$x = 8$$

The number of students who play the three games is 8

$$\text{c) The number of students who play only two games is } 13 + 7 + 9 = 29$$

$$19) \text{ a) } AB = \sqrt{(1 + 5)^2 + (0.5 - 3.5)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$$

$$AC = \sqrt{(-6 + 5)^2 + (-6 - 3.5)^2} = \sqrt{(-1)^2 + (-9.5)^2} = \sqrt{1 + 90.25} = \sqrt{91.25}$$

$$BC = \sqrt{(-6 - 1)^2 + (-6 - 0.5)^2} = \sqrt{(-7)^2 + (-6.5)^2} = \sqrt{49 + 42.25} = \sqrt{91.25}$$

$$\text{b) Since } AC = BC = \sqrt{91.25} \text{ or } AC = BC$$

ABC is an isosceles triangle.

$$\text{c) } PC = \sqrt{(-6 + 2)^2 + (-6 - 2)^2} = \sqrt{(-4)^2 + (-8)^2} = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$$

20) a) On graph paper

b) On graph paper

$$A'(2, -4) \quad B'(6, -4) \quad C'(7, -2)$$

$$A''(-2, 4) \quad B''(-6, 4) \quad C''(-7, 2)$$

c)

